REVIEWS

Non-linear Wave Propagation with Applications to Physics and Magnetohydrodynamics. By A. Jeffrey and T. Taniuti. Academic Press, 1964. 369 pp. £4. 6*s*.

This book is much less general than the title and preface imply. First, it is limited to non-dispersive waves governed by hyperbolic equations. Dispersive waves, such as ocean waves, are not discussed and would, in fact, be ruled out by the definition of wave propagation that is adopted. Secondly, even for non-dispersive waves, it is really a book on the mathematical structure of hyperbolic differential equations. The emphasis is on the basic theory of characteristics and shock conditions, and related topics such as geometrical optics for the propagation of weak discontinuities. The problems are nearly all one-dimensional and refer to simple wave solutions or the resolution of discontinuous initial values by piecing together constant shocks, contact discontinuities and centred rarefaction waves. There is no discussion of topics like shock waves of varying strength, cylindrical and spherical waves, shock wave structure, diffusive effects of dissipation, and so on.

The first half of the book gives the general theory. Waves are introduced as discontinuities in derivatives, and this leads to the theory of characteristics and the definitions of hyperbolic equations. Uniqueness and existence theorems are given with the iteration proofs in the language of contraction mappings in metric spaces. On the reduction of higher-order equations to first-order systems (p. 40) there is one misleading statement. Extra characteristics may arise because the first-order system may have a wider class of solutions than those satisfying the original equation, and the extra characteristics apply to the additional solutions; they should not be excluded because of some mysterious smoothness condition. The geometry of rays and wave fronts is discussed using the defining equation of a characteristic surface as a Hamiltonian, and this is done well. Then the two main chapters are on the theory of characteristics, Riemann invariants, simple waves and shock conditions for a general first-order system in matrix form. Shock waves are introduced via weak solutions of conservation equations. The non-uniqueness of such discontinuous solutions is duly noted, but the non-uniqueness of the possible conservation equations derivable from a given set of equations should also be pointed out. Throughout the book, shock waves are discontinuities. There is no estimate of thickness or comment on the relevance of these discontinuities to the real situation. Viscosity is mentioned briefly in the spirit of a mathematical artifice for resolving non-uniqueness of weak solutions rather than as an important real effect. The physical applications in this first section are little more than calculations of the various characteristic equations and shock conditions for special cases such as Maxwell's equations and the equations of gas dynamics. There is a brief section on the numerical solution of problems but only one method is described and the crucial stability condition for the finite difference scheme is hidden under the phrase 'it is clear that for simplicity we must choose Δx and Δt such that...'.

Although most of the material in this first half can be found in Courant-Hilbert, vol. II, there are some valuable additional topics. The most important is a discussion of the 'evolutionary conditions' for the stability of shock discontinuities. Particularly in magnetohydrodynamics, there are spurious discontinuities satisfying the shock conditions which are not excluded by the requirement of entropy increase. One way of excluding these is to note that they are unstable to small disturbances. Investigation of the perturbed equations shows that a stable solution will be uniquely determined if and only if the number of outgoing waves is one less than the number of shock conditions, provided also that a certain set of vectors arising in the perturbed solution is linearly independent. The exceptional case when one of the wavelets has zero phase velocity gives trouble and, apparently, some results slightly different from those of Polovin. In this discussion of stability it would have been good to include comparison with the alternative requirement of a satisfactory shock structure when dissipative effects are included.

The second half of the book is an application of the general theory to the equations for magnetohydrodynamic waves in an infinitely conducting fluid. The formulae for the characteristic relations, simple wave solutions and shock conditions are all examined in detail. These are applied to interaction problems in which solutions are built up by combinations of fast, slow and transverse shocks and the corresponding rarefaction waves. It all gets very involved and culminates in the hilarious diagrams on pages 284–7. (These are reminiscent of Steinberg's drawings for *The New Yorker*.) It is perhaps good to have all the algebra and enumeration of cases on record, but it does not lead to a good understanding of magnetohydrodynamic waves. It would be far better to have a thorough discussion of some real problem. The assumptions are listed on the first page of this section on magnetohydrodynamics, but there is no sober assessment of when these apply in real situations. In view of the difficulty of attaining high conductivity, some estimate of the thickness of magnetohydrodynamic shocks should have been included, together with some discussion of how the complicated and unusual wave patterns (such as the cusps on the slow waves) are likely to be modified.

To sum up, I think the authors have done a patient and conscientious job on the material they present, but have taken a narrow view of the subject. They dwell on the mathematical formalities of standard theory; there is no real guidance on how to solve difficult non-linear problems, no approximate methods, no comparison with experiment, and no attempts to increase understanding through physical arguments.

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